

For an expression of the form $x^2 + bx$, you can add a constant c to the expression so that the expression $x^2 + bx + c$ is a perfect square trinomial. This process is called **completing the square**.

KEY CONCEPT *For Your Notebook*

Completing the Square

Words To complete the square for the expression $x^2 + bx$, add the square of half the coefficient of the term bx .

Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

10.5 Example 1 Complete the Square

Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

a. $x^2 + 8x + c$ $\left(\frac{b}{2}\right)^2$
 $x^2 + 8x + 16$ $\left(\frac{8}{2}\right)^2$
 $(x+4)^2$ $(4)^2$
 16

b. $x^2 - 5x + c$ $\left(\frac{b}{2}\right)^2$
 $x^2 - 5x + \frac{25}{4}$ $\left(-\frac{5}{2}\right)^2$
 $\left(x - \frac{5}{2}\right)^2$ $\left(\frac{25}{4}\right)$

Example 2- Solving Equations

Solve by Completing the Square.

a. $x^2 - 16x = -15$ $\left(\frac{b}{2}\right)^2$
 $x^2 - 16x + 64 = -15 + 64$ $\left(-\frac{16}{2}\right)^2$
 $\sqrt{(x-8)^2} = \sqrt{49}$ $(-8)^2$
 $x-8 = \pm 7$ 64
 $x = 8 \pm 7$
 $x = 15, 1$

b. $x^2 + 2x = 3$ $\left(\frac{b}{2}\right)^2$
 $x^2 + 2x + 1 = 3 + 1$ $\left(\frac{2}{2}\right)^2$
 $\sqrt{(x+1)^2} = \sqrt{4}$ $(1)^2$
 $x+1 = \pm 2$
 $x = -1 \pm 2$
 $x = 1, -3$

Example 3 - Solve a Quadratic Equation in standard form

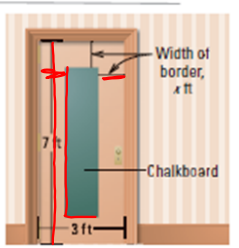
Solve by Completing the Square. Round answers to the nearest hundredth.

a. $m^2 + 10m + 8 = 0$ $\left(\frac{b}{2}\right)^2$
 $m^2 + 10m = -8$ $\left(\frac{10}{2}\right)^2$
 $m^2 + 10m + 25 = -8 + 25$ $(5)^2$
 $\sqrt{(m+5)^2} = \sqrt{17}$ 25
 $m+5 = \pm 4.12$
 $m = -5 \pm 4.12$
 $M = -9.12, -0.88$

b. $3g^2 - 24g + 27 = 0$
 $3g^2 - 24g = -27$
 $\frac{3g^2 - 24g}{3} = \frac{-27}{3}$
 $g^2 - 8g = -9$
 $g = 6.65, 1.35$

Example 4- Story Problems

Crafts You decide to use chalkboard paint to create a chalkboard on a door. You want the chalkboard to have a uniform border as shown. You have enough chalkboard paint to cover 6 square feet. Find the width of the border to the nearest inch.



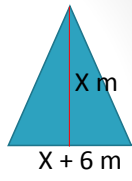
$(7-2x)(3-2x) = 6$ $\left(-\frac{2}{2}\right)^2$
 $21 - 6x - 14x + 4x^2 = 6$ $\left(\frac{2}{2}\right)^2$
 $21 - 20x + 4x^2 = 6$ 6.25
 $\frac{4x^2 - 20x}{4} = \frac{-15}{4}$
 $x^2 - 5x = -3.75$
 $\left(x - \frac{5}{2}\right)^2 = -3.75 + 6.25$
 $\left(x - \frac{5}{2}\right)^2 = 2.5$
 $x - \frac{5}{2} = \pm 1.58$
 $x = 2.5 \pm 1.58$

4.68 (6.92)
 1.16 x 5.16

Another Story Problem

Solve by completing the square.

The given triangle has an area of 108m².



Find the value of x .

$A = \frac{1}{2}bh$ $\left(\frac{b}{2}\right)^2$
 $108 = \frac{1}{2}(x+6)x$ $\left(\frac{6}{2}\right)^2$
 $\pm (108) \left(\frac{1}{2}(x^2 + 6x)\right) \pm 9$
 $216 = x^2 + 6x$
 $216 + 9 = x^2 + 6x + 9$
 $\sqrt{225} = \sqrt{(x+3)^2}$
 $\pm 15 = x + 3$
 -3 -3

$-3 \pm 15 = x$
 $12 = x$
 $x = 12m$